Asymptotic description of flutter amplitude saturation by friction forces

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Carlos Martel[1] (carlos.martel@upm.es) and Roque Corral[1,2]

[1]ETSI Aeronáuticos, UPM, 28040 Madrid, Spain
[2]Industria de Turbopropulsores S.A., 28830 Madrid, Spain

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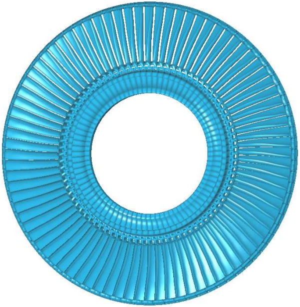
Computation of friction saturated flutter

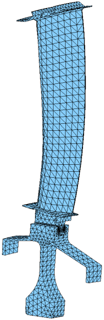
Coupled integration of Fluid NS equations +
Structural elastic motion +
Nonlinear elements at the friction interfaces
Many cycles required to converge, friction numerical stiffness

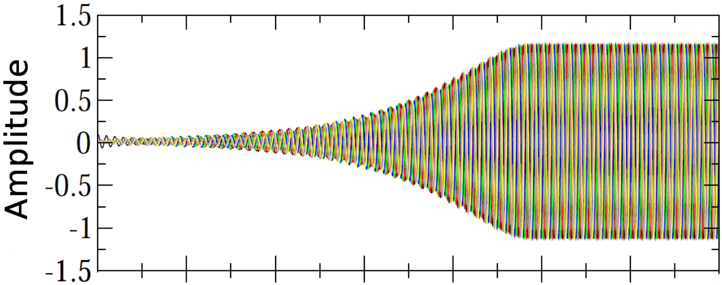
Very CPU costly computation



Coupled Fluid NS equations (linearized) +
Structural elastic motion +
Nonlinear elements at the friction interfaces
Single nonlinear TW: $F(k\theta + \omega t)$ $\rightarrow$reduction to a sector (?)
Many cycles required, friction numerical stiffness

Very CPU costly computation



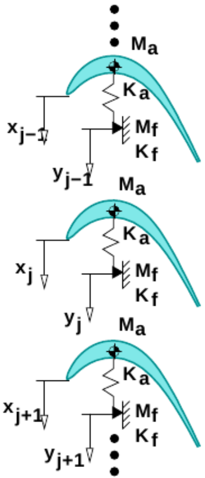


Slow flutter growth and then slow friction saturation

Many fast oscillation cycles below a slow envelope

Asymptotic techniques to filter out the fast oscillation

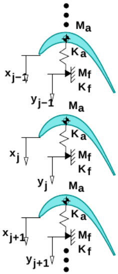
Model bladed disk



$$
\begin{aligned}
m_a \ddot{x}_j + k_a(x_j - y_j) &= F_a(\ldots, x_j, \ldots; \ldots, \dot{x}_j, \ldots) \\
m_f \ddot{y}_j + k_a(y_j - x_j) &= F_f(y_j)
\end{aligned}
$$

Corral & Gallardo, GT2011-45983

Model bladed disk

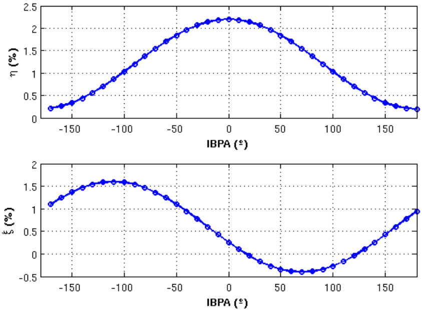


$$
\begin{aligned}
m_a\ddot{x}_j + k_a(x_j - y_j) &= F_a(\dots, x_j, \dots; \dots, \dot{x}_j, \dots) \\
m_f\ddot{y}_j + k_a(y_j - x_j) &= F_f(y_j)
\end{aligned}
$$

$j = 1 \dots N$ sectors (N=36)
2 DOF/sector:
Blade displacement: x_j
Friction (microslip) displacement: y_j
Aero forces F_a, Friction Forces F_f

Aero forces

Linearized aerodynamic forces, TW formulation, couple all sectors

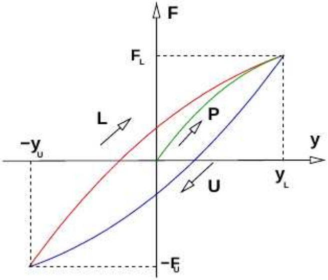


Critical damping ratio ξ vs. ND (unstable from 2 ND to 12 ND)
Frequency correction η vs. ND
Both are small; many cycles required to show their effect.

Friction forces

Sellgren & Olofsson model (1999)

$$F_f(y_j) = F^{U/L} \pm F_c(1 - (\frac{y_j - y^{U/L}}{y_c})^{\frac{5}{2}}),$$



Microslip regime, nonlinear hysteresis loop
Characteristic microslip displacement y_c and force F_c
Local at each sector.

Nondimensional model

$$\begin{aligned} \tilde{x}_{jTT} + \tilde{x}_j - \theta \tilde{y}_j &= F_a/(m_a \omega_a^2 x_c), \\ \delta \theta \tilde{y}_{jTT} + \theta \tilde{y}_j - \tilde{x}_j &= F_f/F_c, \end{aligned}$$

$$\begin{aligned} \tilde{x}_j &= x_j/x_c, \\ \tilde{y}_j &= y_j/y_c, \\ T &= t\omega_a, \end{aligned}$$

y_c characteristic microslip displacement scale
$x_c = F_c/k_a$ characteristic blade displacement
ω_a blade elastic vibration frequency

Model bladed disk

Nondimensional model

$$
\begin{aligned}
\tilde{x}_{jTT} + \tilde{x}_j - \theta \tilde{y}_j &= F_a/(m_a \omega_a^2 x_c) \\
\delta \theta \tilde{y}_{jTT} + \theta \tilde{y}_j - \tilde{x}_j &= F_f/F_c
\end{aligned}
$$

Asymptotic limit:

$$
\theta = \frac{y_c}{x_c} = \frac{k_a}{(F_c/y_c)} \ll 1
$$

$$
\delta = \frac{m_f}{m_a} \ll 1
$$

$$
\frac{F_a}{m_a \omega_a^2 x_c} \ll 1
$$

$$
\frac{F_f}{F_c} \sim 1
$$

Nondimensional model

$$
\begin{aligned}
\tilde{x}_{jTT} + \tilde{x}_j - \theta \tilde{y}_j &= F_a/(m_a \omega_a^2 x_c) \\
\delta \theta \tilde{y}_{jTT} + \theta \tilde{y}_j - \tilde{x}_j &= F_f / F_c
\end{aligned}
$$

Asymptotic multiple scales method: $T \sim 1, \quad \tau = T\theta \gg \mathbf{1}$

$$
\begin{aligned}
\tilde{x}_j &= \tilde{x}_j^0(T,\tau) + \theta \tilde{x}_j^1(T,\tau) + \ldots \\
\tilde{y}_j &= \tilde{y}_j^0(T,\tau) + \theta \tilde{y}_j^1(T,\tau) + \ldots
\end{aligned}
$$

1st order

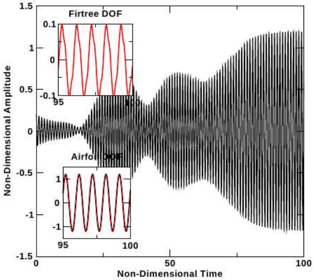
$$
\begin{aligned}
\tilde{x}_{jTT}^0 + \tilde{x}_j^0 &= 0 \\
-\tilde{x}_j^0 &= F_f(\tilde{y}_j^0)/F_c
\end{aligned}
$$

1st order

$$\begin{aligned}\tilde{x}^0_{jTT} + \tilde{x}^0_j &= 0\\ -\tilde{x}^0_j &= F_f(\tilde{y}^0_j)/F_c\end{aligned}$$

Fast time dynamics

$$\begin{aligned}\tilde{x}^0_j(T,\tau) &= X_j(\tau)e^{iT} + \text{c.c.}\\ \tilde{y}^0_j(T,\tau) &= P(T,X_j) = \sum_{k=1}^{\infty} P_k(|X_j|)e^{ik(T+\phi_j)} + \text{c.c.},\end{aligned}$$



Corral & Gallardo, GT2008-51416

2nd order

$$\tilde{x}^1_{jTT} + \tilde{x}^1_j = -2\frac{\partial^2 \tilde{x}^0_j}{\partial\tau\partial T} + \tilde{y}^0_j + \frac{F_a(\tilde{x}^0_1,\ldots,\tilde{x}^0_N;\tilde{x}^0_1,\ldots,\tilde{x}^0_N)/(m_a\omega_a^2 x_c)}{\theta}$$

Bounded dynamics in the fast scale

$$2i\frac{dX_j}{d\tau} = P_1(|X_j|)e^{i\phi_k} + \frac{L_a(X_1,\ldots,X_N)/(m_a\omega_a^2 x_c)}{\theta}$$

Only one component has an effect on the slow time dynamics

$$\tilde{y}^0_j(T,\tau) = P(T,X_j) = \sum_{k=1}^{\infty} P_k(|X_j|)e^{ik(T+\phi_j)} + \text{c.c.},$$

$$P_1(|X_j|)e^{i\phi_k} = \frac{1}{2\pi}\int_0^{2\pi} \tilde{y}^0_j e^{-iT}\, dT$$

Displacement formulation: $X_1, \dots X_N$

$$2i\frac{d}{d\tau}\begin{bmatrix} X_1 \\ \vdots \\ X_j \\ \vdots \\ X_N \end{bmatrix} = \begin{bmatrix} Q(|X_1|) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & Q(|X_N|) \end{bmatrix} \begin{bmatrix} X_1 \\ \vdots \\ X_j \\ \vdots \\ X_N \end{bmatrix}$$

$$-2c_a\mathbf{P}\begin{bmatrix} \tilde{\eta}_1 + i\tilde{\xi}_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \tilde{\eta}_N + i\tilde{\xi}_N \end{bmatrix}\mathbf{P}^H \begin{bmatrix} X_1 \\ \vdots \\ X_j \\ \vdots \\ X_N \end{bmatrix}.$$

$P(|X|) = Q(|X|)|X|, \; Q \in \mathbb{C}$
$c_a = (F_a/(m_a\omega_a^2 x_c))/\theta$

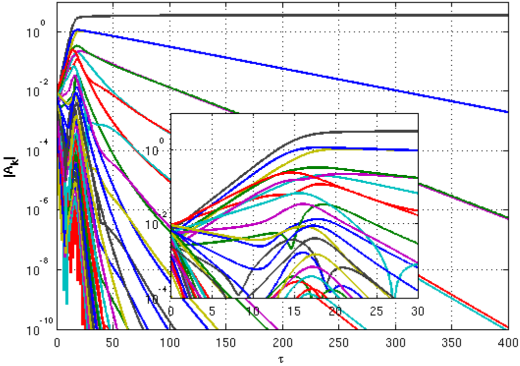
TW formulation: $A_1, \dots A_N$

$$2i\frac{d}{d\tau}\begin{bmatrix} A_1 \\ \vdots \\ A_k \\ \vdots \\ A_N \end{bmatrix} = \mathbf{P}^H \begin{bmatrix} Q(|X_1|) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & Q(|X_N|) \end{bmatrix} \mathbf{P} \begin{bmatrix} A_1 \\ \vdots \\ A_k \\ \vdots \\ A_N \end{bmatrix}$$

$$-2c_a \begin{bmatrix} \tilde{\eta}_1 + i\tilde{\xi}_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \tilde{\eta}_N + i\tilde{\xi}_N \end{bmatrix} \begin{bmatrix} A_1 \\ \vdots \\ A_k \\ \vdots \\ A_N \end{bmatrix}.$$

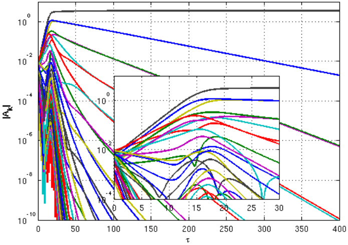
Nonlinear TWs exist, for the linearly unstable ones.

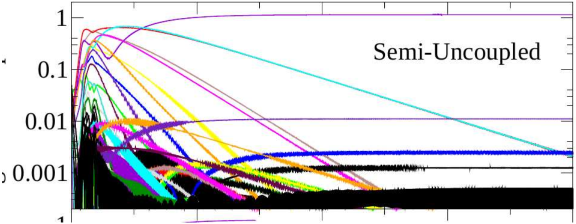
Random initial conditions of size $\sim .01$, $c_a = 1$, $N = 36$
$Q(|X|)$ from the S&O model



Final state: nonlinear saturated k=7 TW (most unstable)

Results

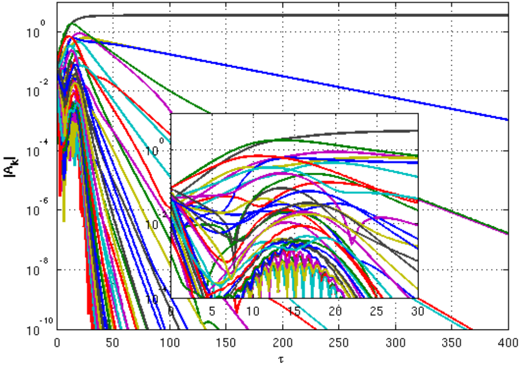




Corral & Gallardo, GT2011-45983

Results

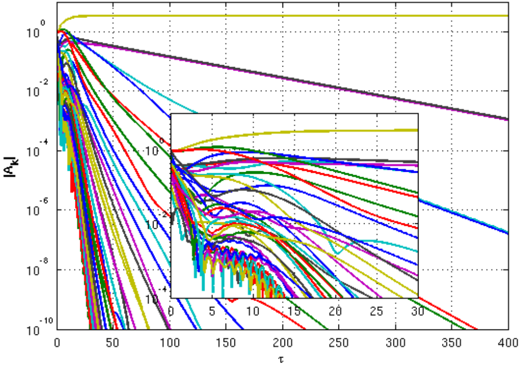
Random initial conditions of size ~ 1, $c_a = 1$, $N = 36$
$Q(|X|)$ from the S&O model



Final state: nonlinear saturated k=7 TW (most unstable)

Results

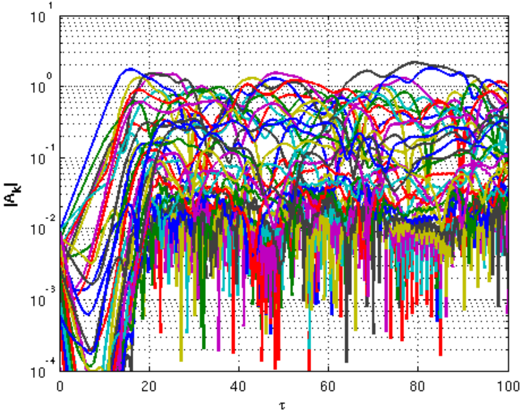
Random initial conditions of size ~ 10, $c_a = 1$, $N = 36$
$Q(|X|)$ from the S&O model



Final state: nonlinear saturated k=6 TW (second most unstable)

Results

Random initial conditions of size $\sim 10^{-2}$, $c_a = 1$, $N = 36$
$Q(|X|)$ from the S&O model but with $10 \times \Re(Q)$



No final state as a saturated TW (not realistic).

Final Remarks

- Simplified description, asymptotically accurate, TWs with frequencies near the unstable ones.
- Elastic oscillation filtered out. Numerical problems from the even shorter time scales associated with nonlinear friction models also removed.
- Can be applied to detailed model with nonlinear friction. (aero, friction $<<$ elastic)
- Sequence: Purely elastic TW modes $\rightarrow$ Microslip displacements $\rightarrow$ friction and aero long time-scale effects
- The effect of the friction is reduced to a single coefficient. Exploration of different friction models (Schwingshackl, Petrov & Ewins, JEGTP, 2012).
- **NEXT:** nonlinear TW, stability analysis, always the final state?
- **NEXT:** inclusion of FR
- **NEXT:** inclusion of mistuning